

# One dimensional $s$ -wave holographic superconductor with supercurrent

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**ABSTRACT:** We study the one dimensional  $s$ -wave holographic superconductor by turning on the vector potential  $A_x$  in the bulk, which behaves as  $A_x = A_x^{(0)} \ln z + A_x^{(1)}$  on the boundary. By solving the model with fixed  $A_x^{(0)}$ , we find that if we identify the  $A_x^{(0)}$  with the supercurrent  $j_x$  of the holographic superconductor, the results agree with the Ginzburg-Landau theory. For example,  $A_x^{(0)}$  will break the superconductivity, and the critical value of  $A_x^{(0)}$  is proportional to  $(T_c - T)^{3/2}$ .

**KEYWORDS:** Holography, Superconductivity, AdS<sub>3</sub>/CFT<sub>2</sub>.

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## Contents

<b>1. Introduction</b>	<b>1</b>
<b>2. The condensed phase with <math>A_x = 0</math></b>	<b>2</b>
2.1 The Einstein-Maxwell-charged scalar system in $AdS_3$ black hole	2
2.2 Phase diagram and the free energy analysis	4
<b>3. The superconductor with fixed <math>A_x^{(0)}</math></b>	<b>5</b>
3.1 The condensate versus temperature	5
3.2 Results that agree with the G-L theory	6
3.2.1 Current via the superfluid velocity	7
3.2.2 The critical current via temperature	8
<b>4. The superconductor with fixed <math>A_x^{(1)}</math></b>	<b>8</b>
<b>5. Conclusion</b>	<b>9</b>

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## 1. Introduction

The AdS/CFT correspondence [1–4] has been taken as a very useful method to study strongly coupled phenomena now. A successful application of the AdS/CFT correspondence to condensed matter physics is to study the superconductors. The strategy to build a holographic superconductor is to couple an  $AdS$  black hole with some charged field and the  $U(1)$  gauge fields. The black hole will have hair when the temperature of the black hole is low enough. The charged hair breaks the  $U(1)$  gauge symmetry in the bulk and the black hole is superconducting [5]. By putting a charged scalar field coupled to the  $U(1)$  gauge field in  $AdS_4$  black hole, one can get the  $s$ -wave holographic superconductor in which the order parameter is a scalar [6, 7]. By putting a pure  $SU(2)$  gauge field in the bulk theory, one can get the  $p$ -wave holographic superconductor with a vector order parameter [8–12]. In order to build a  $d$ -wave holographic superconductor we need a charged tensor field coupled to a  $U(1)$  gauge field in the bulk that leads to a tensor order parameter [13–15].

Most of the studies on holographic superconductors have focused on the  $(2+1)$  dimensional systems by using  $AdS_4/CFT_3$ . The  $(3+1)$  dimensional holographic superconductor shows similar properties as the  $(2+1)$  dimensional superconductors, as studied in [16] by using  $AdS_5/CFT_4$ . The  $(1+1)$  dimensional  $s$ -wave and  $p$ -wave holographic superconductors were explored in [17] and [18] respectively by using  $AdS_3/CFT_2$ . Due to the logarithm behavior of  $A_x$  on the boundary, we have to realize that the  $AdS_3$  realization of holographic superconductor is an honest superconductor with a dynamic gauge field on

the boundary, in which the local boundary symmetry is spontaneously broken [18]. The holographic superconductors with dynamic gauge fields in higher dimensions were studied in [19–22]. A holographic quantum liquid in (1+1) dimensions from a probe D3 brane in the AdS Schwarzschild planar black hole background was studied in [23].

In this paper, we studied the one dimensional  $s$ -wave holographic superconductor by switching on the vector potential  $A_x$  in the bulk theory.  $A_x$  behaves as  $A_x = A_x^{(0)} \ln z + A_x^{(1)}$  on the  $AdS_3$  boundary. By solving the model with fixed  $A_x^{(0)}$ , we find that  $A_x^{(0)}$  plays the role of supercurrent in a superconductor. If we identify  $A_x^{(0)}$  with the supercurrent  $j_x$  and  $-A_x^{(1)}$  as the superfluid velocity  $v_x$ , the results agree with the Ginzburg- Landau (G-L) theory for a superconductor with supercurrent. For example,  $A_x^{(0)}$  will break the superconductivity, the critical value of  $A_x^{(0)}$  being proportional to  $(T_c - T)^{3/2}$ . These results indicate that we can interpret the source term  $A_x^{(0)}$  as the supercurrent  $j_x$  while  $-A_x^{(1)}$  as the superfluid velocity  $v_x$ . The G-L theory results we found in (1 + 1) dimensional  $s$ -wave holographic superconductors have also been founded in the (2 + 1) dimensional holographic superconductors with supercurrent [24]. In analogy, for the scalar potential  $A_t = A_t^{(0)} \ln z + A_t^{(1)}$  on the boundary,  $-A_t^{(0)}$  can be interpreted as the charge density  $\rho$  while  $A_t^{(1)}$  as the chemical potential  $\mu$ .

The organization of this paper is as follows. In Section 2 we first introduce the Einstein-Maxwell-charged scalar system in  $AdS_3$  black hole, and then confirm that there is a continuous phase transition when  $A_x = 0$  by free energy calculations. In Section 3 we solve the system with fixed value of  $A_x^{(0)}$  and we find that  $A_x^{(0)}$  should be the supercurrent of the holographic superconductor. We also try to solve the system with fixed value of  $A_x^{(1)}$  in Section 4. Discussions and conclusions are given in Section 5.

## 2. The condensed phase with $A_x = 0$

In this section we firstly review the results in [17]. We also compute the free energy to confirm the existence of the continuous superconducting phase transition.

### 2.1 The Einstein-Maxwell-charged scalar system in $AdS_3$ black hole

The Lagrangian of the Einstein-Maxwell-charged scalar system reads

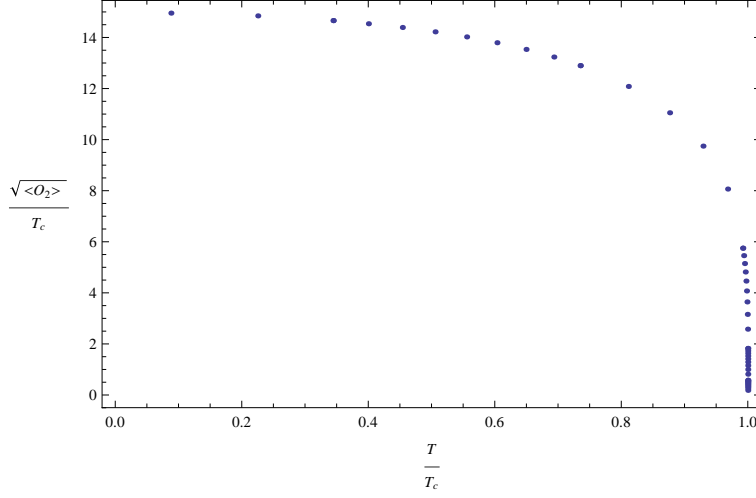
$$\mathcal{L} = -\frac{1}{4}F^{ab}F_{ab} - \frac{1}{2}m^2|\Psi|^2 - |(\partial_\mu - iA_\mu)\Psi|^2. \quad (2.1)$$

The neutral  $AdS_3$  black hole background in Poincaré coordinates is given by [17]

$$ds^2 = \frac{L^2}{z^2} \left( -f(z)dt^2 + dx^2 + \frac{dz^2}{f(z)} \right), \quad (2.2)$$

in which  $f(z) = 1 - z^2$ ,  $z = r_+/r$ , and  $r_+$  is the horizon of the black hole. We set  $L = 1$  and  $r_+ = 1$ . The black hole temperature is given by

$$T = \frac{r_+}{2\pi}. \quad (2.3)$$



**Figure 1:** The condensate versus temperature when  $A_x$  is zero.

With the ansatz

$$\Psi = \Psi(r), A_t = \Phi(r), A_x = A_x(r), \quad (2.4)$$

we have the equations of motion (EOMs) for the three fields,

$$\Psi'' - \frac{2z}{1-z^2}\Psi' - \frac{1}{z}\Psi' + \frac{\Phi^2\Psi}{(1-z)^2} - \frac{A_x^2\Psi}{1-z^2} - \frac{m^2\Psi}{z^2 f} = 0, \quad (2.5)$$

$$\Phi'' + \frac{1}{z}\Phi' - \frac{2\Phi\Psi^2}{z^2(1-z^2)} = 0, \quad (2.6)$$

$$A_x'' - \frac{2z}{1-z^2}A_x' + \frac{A_x'}{z} - \frac{2A_x\Psi^2}{z^2(1-z^2)} = 0. \quad (2.7)$$

The boundary behavior for the two Maxwell fields  $\Phi$  and  $A_x$  at  $z = 0$  are

$$\Phi = \Phi^{(0)} \ln(\Lambda z) + \Phi^{(1)} + \dots, \quad (2.8)$$

$$A_x = A_x^{(0)} \ln(\Lambda z) + A_x^{(1)} + \dots, \quad (2.9)$$

where  $\Lambda$  is the renormalization scale included in the logarithm. We set  $\Lambda$  to be 1 without loss of generality. It has been found that there is only one way to quantize the theory (alternative quantization) by defining the coefficients of the logarithmic terms to be the sources [25–27].

As for the scalar field  $\Psi$ , we can only consider the case of  $m^2 = 0$ , since the results for other values of  $m^2$  are similar [17]. The asymptotic behavior of  $\Psi$  near the boundary for  $m^2 = 0$  reads

$$\Psi = \tilde{\Psi}^{(1)} + \tilde{\Psi}^{(2)} z^2 + \dots. \quad (2.10)$$

We can only choose the scalar operator as  $\langle O_2 \rangle = \tilde{\Psi}^{(2)}$ , with the boundary condition  $\tilde{\Psi}^{(1)} = 0$  [17].  $\langle O_2 \rangle$  is the order parameter of the superconductor. On the horizon, regularity requires  $\Phi = 0$ , with the other two fields  $\Psi$  and  $A_x$  being finite. In this section we set  $A_x = 0$  and solve the two EOMs Eq. (2.5) and Eq. (2.6) for  $\Phi$  and  $\Psi$  with vanishing  $A_x$ .

## 2.2 Phase diagram and the free energy analysis

The phase diagram of the superconductor with  $A_x = 0$  is plotted in Fig. 1. It is clear that there is a continuous phase transition at  $T_c$ . After computing the free energy density we will confirm the result further.

According to the AdS/CFT dictionary, the free energy of the dual field theory is related to the on-shell action of the classical gravity theory,

$$\mathcal{F} = -TS_{os} + \dots \quad (2.11)$$

where the ellipsis denotes boundary terms we may need. Employing the equations of motion, the on-shell action can be written as

$$S_{os} = \int dx^2 \left( \frac{z}{2} \Phi \Phi' - \frac{zf}{2} A_x A'_x - \frac{f}{z} \Psi \Psi' \right) \Big|_{z=0} + \int dx^3 \left( \frac{A_x^2 \Psi^2}{z} - \frac{A_t^2 \Psi^2}{zf} \right). \quad (2.12)$$

By plugging in the asymptotic behaviors of  $\Phi$ ,  $A_x$  and  $\Psi$  we get

$$S_{os} = \int dx^2 \left( \frac{1}{2} \Phi^{(0)} (\Phi^{(1)} + \Phi^{(0)} \ln z) - \frac{1}{2} A_x^{(0)} (A_x^{(1)} + A_x^{(0)} \ln z) \right) \Big|_{z=0} + \int dx^3 \left( \frac{A_x^2 \Psi^2}{z} - \frac{A_t^2 \Psi^2}{zf} \right). \quad (2.13)$$

In order to work in the ensemble with fixed  $A_i^{(0)}$ , we need to add the boundary [25, 26]

$$S_{bdy} = \frac{1}{2} \int dt dx \left( \sqrt{-h} A_\mu F^{z\mu} \right) \Big|_{z=0}, \quad (2.14)$$

where  $h$  is the induced metric of the boundary. We can see  $S_{os} + S_{bdy}$  which is divergent. In order to cancel the divergence, we add the counter term [26]

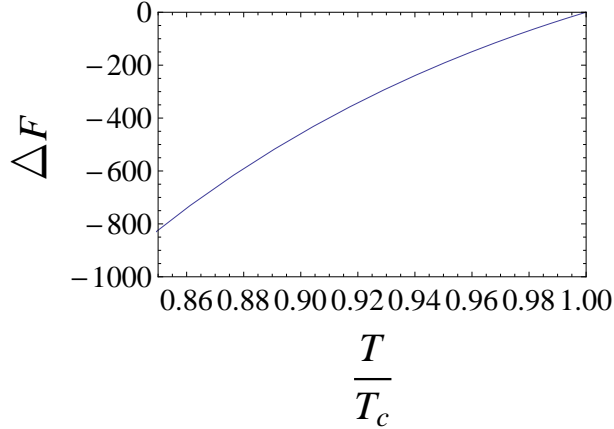
$$S_{CT} = \int dx^2 \left( \sqrt{-h} F_{zt} F^{zt} \ln z + \sqrt{-h} F_{zx} F^{zx} \ln z \right) \Big|_{z=0}, \quad (2.15)$$

Finally, the free energy density for the superconductor reads

$$F = \frac{\mathcal{F}}{l} = -\Phi^{(0)} \Phi^{(1)} + A_x^{(0)} A_x^{(1)} - \int dz \left( \frac{A_x^2 \Psi^2}{z} - \frac{A_t^2 \Psi^2}{zf} \right), \quad (2.16)$$

where  $l$  is the length of the superconducting wire. The superconductor is uniform since the order parameter is independent of  $x$ .

In Fig. 2 we plot the free energy density difference between the superconducting and normal states. It can be seen that the superconducting state is more stable than the normal state when  $T < T_c$ .



**Figure 2:** The free energy difference between the superconducting state and the normal state with  $A_x = 0$ . It is clear that the superconducting state is stabler when  $T < T_c$ .

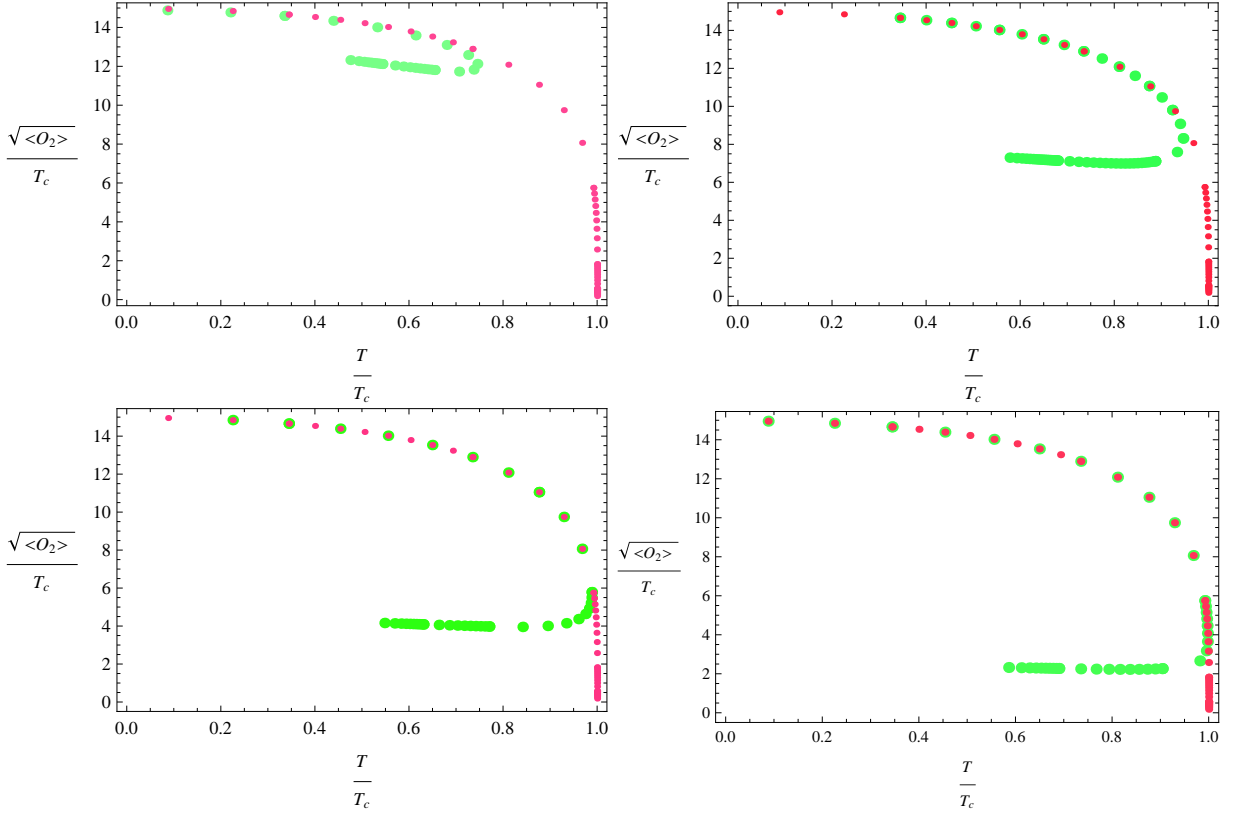
### 3. The superconductor with fixed $A_x^{(0)}$

In  $AdS_4$  setup of holographic superconductor,  $A_x = A_x^{(0)} + A_x^{(1)}z$  on the boundary. It is clear now we can interpret  $-A_x^{(0)}$  as the superfluid velocity and  $A_x^{(1)}$  as the supercurrent. There are two kinds of quantizations for  $A_x$ , which means that we can fix the value of  $A_x^{(0)}$  and let  $A_x^{(1)}$  free to fluctuate, or fix the value of  $A_x^{(1)}$  and let  $A_x^{(0)}$  free to fluctuate. The two methods to solve the problem correspond to experiments where the current, or the superfluid velocity is kept fixed. It has been shown [24, 28–30] that by fixing  $A_x^{(0)}$  or  $A_x^{(1)}$  on the boundary, the  $AdS_4$  black hole holographic superconductors (both  $s$ -wave and  $p$ -wave holographic superconductors) with fixed supercurrent or fixed superfluidity velocity have the same results as G-L theory.

However, since the gauge fields behave much differently on  $AdS_3$  boundary, it is very interesting to ask what will happen when the superconducting black hole has non-vanishing vector potential  $A_x$ . In this section we solve the EOMs with fixed value of  $A_x^{(0)}$  in Eq. (2.9). In the next section we also try to solve the EOMs with fixed value of  $A_x^{(1)}$  in Eq. (2.9).

#### 3.1 The condensate versus temperature

The first important problem to study is how the order parameter changes with the temperature for this holographic superconductor with fixed  $A_x^{(0)}$ . From now on we suppose  $A_x^{(0)}$  is the supercurrent  $j_x$  and  $A_x^{(1)}$  is the superfluid velocity  $-v_x$ . From Fig. 3 it can be seen that when the supercurrent is not zero, there are two solutions of the order parameter corresponding to a fixed temperature. We also show that the solution with lower value of the order parameter takes a larger free energy than the solution with larger values of the order parameter and therefore it is unfavorable. In Fig. 4 we present the free energy of a fixed current  $j_x = 1/100$  for the two branches of solution. It can be obviously seen that the solution with a larger value of the order parameter has a lower free energy. The critical temperature decreases when the current increases, which indicates that there

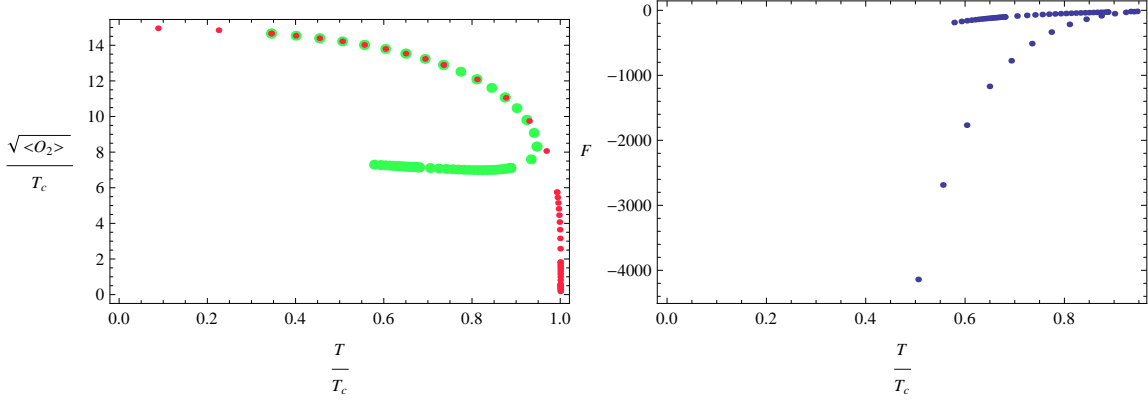


**Figure 3:** Plot of the order parameter versus the temperature for different fixed values of current (The  $A_x^{(0)}$ ). We show three different plots of order parameter versus the temperature for  $j_x=1/10, 1/100, 1/1000, 1/10000$  (from the left to the right). In which we also plot the order parameter versus temperature when  $A_x = 0$  for compare. We have identified  $A_x^{(0)}$  with  $j_x$  already in the plots. (The green plots are the condensate with fixed  $A_x^{(0)}$ , the red plots represent the condensate with  $A_x = 0$ .)

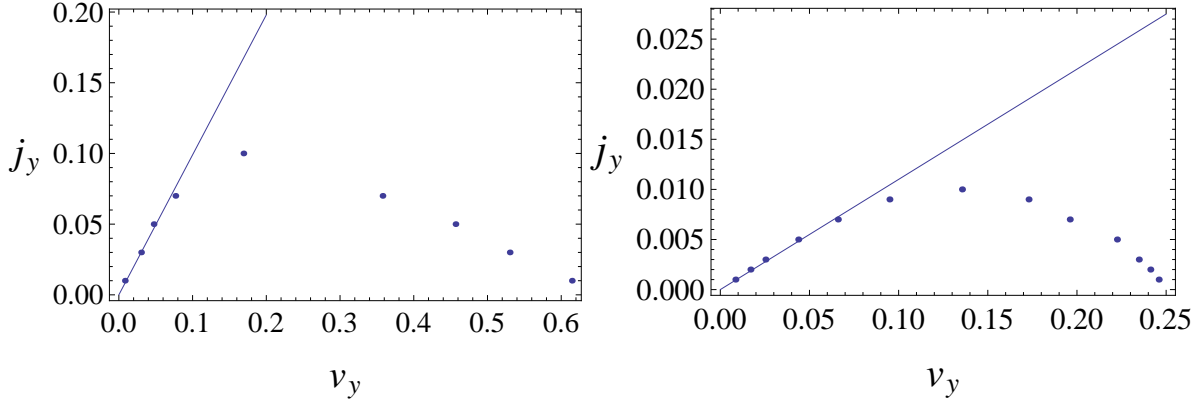
should exist a critical current above which there is no superconductivity. When one lowers the temperature from above the critical temperature, the order of phase transition at the critical temperature for a fixed current should be of first order, since the order parameter jumps from zero to a finite value at the critical temperature. Such a jump will certainly change the energy and so requires some latent heat, which implies that the phase transition should be of first order. This conclusion is the same as the one we shall give by observing the curve of the current  $j_x$  versus the superfluid velocity  $v_x$  at a fixed temperature soon. For the  $AdS_4$   $s$ -wave holographic superconductor with current, the order parameter is also bivalued, and the states with lower value of the condensate have a larger free energy than their counterparts with larger values of the condensate at the same temperature. [24]

### 3.2 Results that agree with the G-L theory

In this section we give more evidences that  $A_x^{(0)}$  behaves the same as a supercurrent in a superconductor. These results indicate that we should interpret  $A_x^{(0)}$  as the supercurrent  $j_x$  in the boundary theory.



**Figure 4:** The free energy of the two branches of solution when  $j_x = 1/100$ , the upper dotted line corresponds to the points with a lower value of the condensate at a given temperature. The left panel shows the corresponding plot of the condensation versus  $T$ . It can be seen that the lower branch solution corresponds indeed to states with larger free energy and is thus metastable.



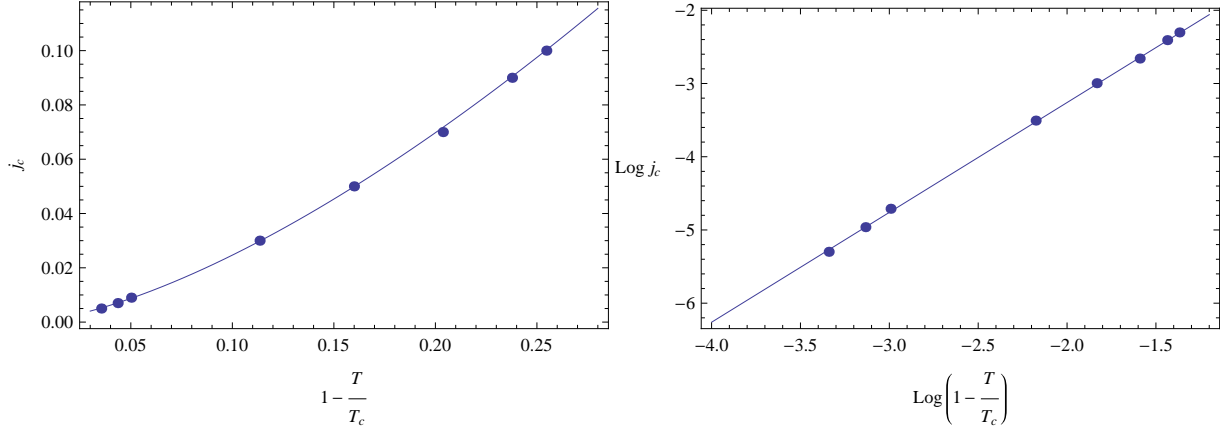
**Figure 5:** Plots of the current  $j_x$  versus the superfluid velocity  $v_x$  at a fixed temperature. The two panels above correspond to  $T/T_c = 0.745385, 0.946225$  (from left to right), at which the critical currents  $j_c$  are  $1/10$  and  $1/100$ , respectively. It is clear that the  $v_x$  and  $j_x$  have a linear relationship when  $j_x$  is not very large.

### 3.2.1 Current via the superfluid velocity

Another physical property by which one can compare the difference between the gravity model of superconductor and the G-L theory is the relation between the current and the superfluidity velocity at a fixed temperature. From this relation we can also get the information of the phase transition at the critical current.

The two plots in Fig. 5 correspond to the two temperatures  $T/T_c = 0.745385$  and  $0.946225$ . For these two temperatures the critical currents are  $j_c = 1/10$  and  $j_c = 1/100$ , respectively. It can be clearly seen that at the temperatures close to  $T_c$ , where the G-L theory works very well, the plots of  $j_x$  versus  $v_x$  are the same as that of G-L theory, in which the  $j_x$  and  $v_x$  have a linear relationship when  $j_x$  is much smaller than  $j_c$ . From these plots we can also infer the order of the phase transitions at critical current or critical





**Figure 6:** Plot of the critical current versus the temperature. The right panel shows a log-log plot from which we can read off the critical exponent, getting 1.499, which agrees with the expected G-L scaling of  $3/2$  within numerical precision. The left panel shows the  $j_c$  versus  $(1 - T/T_c)$ , the solid line is  $0.78(1 - T/T_c)^{3/2}$ .

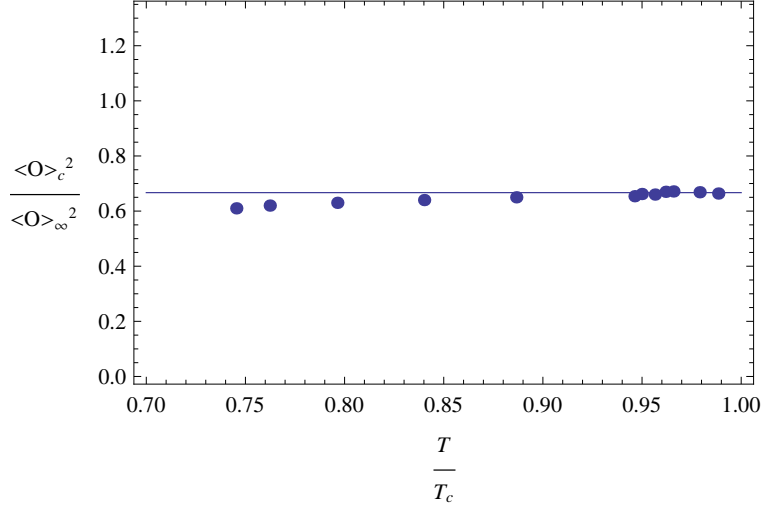
velocity. For the two plots, the maximal velocity corresponds to a vanishing current, which means that the phase transitions at critical velocities the  $v_c = 0.65$  and  $v_c = 0.25$  are of second order. However, the maximal current corresponds to a non-vanishing velocity, which means that the phase transitions at critical currents  $j_c = 1/10$  and  $j_c = 1/100$  are of first order.

### 3.2.2 The critical current via temperature

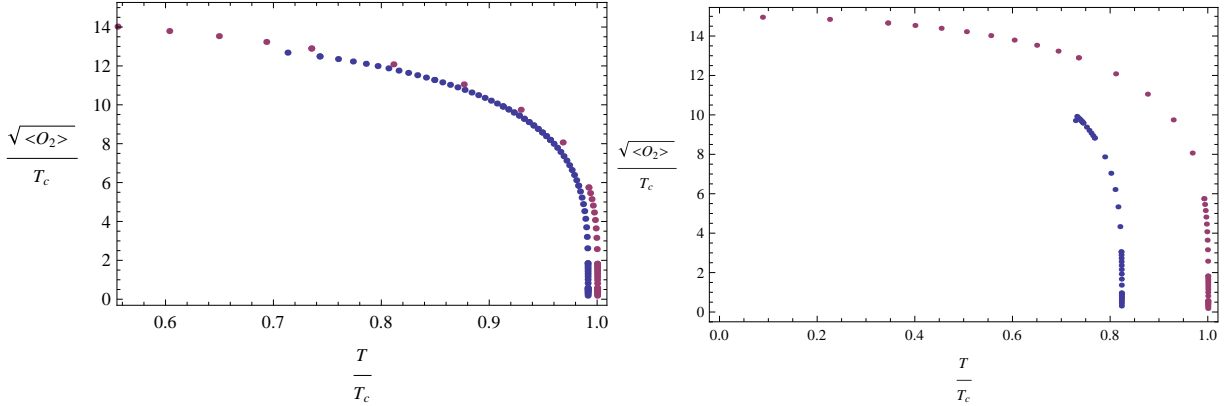
In this subsection we study the critical current  $j_c$  for different  $T$  near  $T_c$  to compare the results with those of the G-L theory. As predicted by G-L theory,  $j_c$  is proportional to  $(T_c - T)^{3/2}$  when the temperature is close to  $T_c$ . As illustrated in Fig. 6, this scaling behavior is indeed obeyed by holographic superconductors for temperatures close to  $T_c$ , which is also the case in the  $(2 + 1)$  dimensional  $s$ -wave model [24]. Another prediction of the G-L theory is that, at any fixed temperature, the norm of the condensate decreases monotonically with respect to the velocity from its maximal value. And the maximal value  $\sqrt{\langle O_2 \rangle_\infty}$  corresponds to zero velocity and zero current. As shown in Fig. 5, the critical current is reached before the velocity reaches its maximal value. The norm of the condensate has an intermediate value  $\sqrt{\langle O_2 \rangle_c}$  at the maximal current. The G-L theory tells us that the ratio of  $\sqrt{\langle O_2 \rangle_c}$  to  $\sqrt{\langle O_2 \rangle_\infty}$  is exactly  $2/3$ . From Fig. 7 it can be seen that this is indeed the case for the  $(1 + 1)$  dimensional  $s$ -wave holographic superconductor.

## 4. The superconductor with fixed $A_x^{(1)}$

Before we give the results when we solve the EOMs with fixed  $A_x^{(1)}$ . We would like to emphasize that there is only one kind of quantization of the theory, we can only take  $A_x^{(0)}$  is the source. However, fixing  $A_x^{(1)}$  does not contradict the statement in [25], we are simply in another ensemble which can be changed to the ensemble in last section with fixed  $A_x^{(1)}$  by a Legendre transformation [26]. From the numerical computation we find that to solve the



**Figure 7:** Plot of the ratio  $(\sqrt{\langle O_2 \rangle_c} / \sqrt{\langle O_2 \rangle_\infty})^2$  versus the temperature. The solid line corresponds to the value of  $2/3$  predicted by the G-L theory and it also appears in the  $AdS_4$   $s$ -wave model.



**Figure 8:** Plot of the order parameter versus the temperature for different fixed values of velocity (The  $-A_x^{(1)}$ ). We show three different plots of order parameter versus the temperature for  $-A_x^{(1)} = 1/10, 1/2$  (from the left to the right). It can be seen that the phase transition is of second order. (The dark blue plots are the condensate with fixed  $A_x$ , the dark red plots represent the condensate without  $A_x$ .)

EOMs with fixed  $A_x^{(0)}$  is indeed much more difficult numerically, it will take very long time for the shooting method to give a solution. In Fig. 7 we give two samples of solutions for fixed  $A_x^{(1)} = -0.1$  and  $-0.5$  respectively. It can be seen that the phase transition with the small values of velocity is of second order, which confirms our discussion in section 3.2.1.

## 5. Conclusion

With the fact the Maxwell gauge fields  $A_t$  and  $A_x$  behave as in Eq. (2.8) and Eq. (2.9) on the  $AdS_3$  boundary, and an important question arises as how to interpret the coefficients  $A_i^{(0)}$  and  $A_i^{(1)}$ . Formally, this question was already answered in [18, 25, 26], the source term

$A_t^{(0)}$  is the charge density and  $A_t^{(1)}$  is the chemical potential. In this paper we have confirmed the conclusion given in [18, 25, 26] by studying the properties of the  $(1 + 1)$  dimensional holographic superconductor with nonzero  $A_x$ . By solving the EOMs we found that if we interpret  $A_x^{(0)}$  and  $-A_x^{(1)}$  in Eq. (2.9) as the supercurrent  $j_x$  and the superfluidity velocity  $v_x$  respectively, the results near the critical temperature agree qualitatively with several properties of the Ginzburg-Landau theory:

- The phase transition with supercurrent is of first order
- The critical supercurrent  $j_c$  is proportional to  $((T_c - T)^{3/2})$
- The relation between velocity and supercurrent is linear almost all the way up to a given maximum velocity
- The squared ratio of the maximal condensate to the minimal condensate is equal to two thirds

Furthermore, the  $\Phi^{(0)}$  in Eq. (2.8) is the charge density and  $\Phi^{(1)}$  is the chemical potential. We summarize these results as

$$\Phi = \mu - \rho \ln z, \quad (5.1)$$

and

$$A_x = -v_x + j_x \ln z \quad (5.2)$$

on the boundary, where  $\mu$  is the chemical potential,  $\rho$  is the charge density,  $v_x$  is the superfluid velocity and  $j_x$  is the supercurrent.

We state that there should be a supercurrent in the  $(1 + 1)$  dimensional holographic superconductor when we include  $A_x$  in the bulk theory, which corresponds to the coefficient of the logarithmic term on the boundary (see Eq. (2.9)). The existence of supercurrent in a holographic superconductor means there should also be superconducting tunneling effect by building a holographic Josephson Junction. [31–34] Then another way to confirm the conclusion that  $A_x^{(0)}$  is the supercurrent is to find the holographic Josephson Junction in the  $AdS_3$  setup of superconductors. We leave this for future work.

## Acknowledgements

It is a pleasure to thank Xin Gao, Matthias Kaminski, Hai-Qing Zhang and Zhe-Yong Fan for valuable discussions. H. B. Zeng is supported by the Fundamental Research Funds for the Central Universities (Grant No. 1107020117) and the China Postdoctoral Science Foundation (Grant No. 20100481120).

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